A Crop Model and Fuzzy Rule Based Approach for Optimizing Maize Planting Dates in Burkina Faso, West Africa

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ABSTRACT

In sub-Saharan Africa, with its high rainfall variability and limited irrigation options, the crop planting date is a crucial tactical decision for farmers and therefore a major concern in agricultural decision making. To support decision making in rainfed agriculture, a new approach has been developed to optimize crop planting date. The General Large-Area Model for Annual Crops (GLAM) has been used for the first time to simulate maize yields in West Africa. It is used in combination with fuzzy logic rules to give more flexibility in crop planting date computation when compared with binary logic methods. A genetic algorithm is applied to calibrate the crop model and to optimize the planting dates at the end. The process for optimizing planting dates results in an ensemble of optimized planting rules. This principle of ensemble members leads to a time window of optimized planting dates for a single year and thereby potentially increases the willingness of farmers to adopt this approach. The optimized planting date (OPD) approach is compared with two well-established methods in sub-Saharan Africa. The results suggest earlier planting dates across Burkina Faso, ranging from 10 to 20 days for the northern and central part and less than 10 days for the southern part. With respect to the potential yields, the OPD approach indicates that an average increase in maize potential yield of around 20% could be obtained in water-limited regions in Burkina Faso. The implementation of the presented approach in agricultural decision support is expected to have the potential to improve agricultural risk management in these regions dominated by rainfed agriculture and characterized by high rainfall variability.

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1. Introduction

Rainfed agriculture in sub-Saharan Africa (SSA) is characterized by prolonged dry spells, droughts, and low inputs of manures, chemical fertilizers, and insecticides. Farmers still suffer from low productivity. Nevertheless, this agricultural system remains the dominant source of staple food production and the livelihood foundation for SSA countries. Several studies addressing the specific agricultural problems have shown that SSA is a water-scarce region (Challinor et al. 2007; Roudier et al. 2011; Biazin et al. 2012), where farmers have to cope with high rainfall variability. Different soil and water management techniques have been developed and promoted throughout SSA countries to optimize water consumption by plants (Rockstrom et al. 2002; Kaboré and Reij 2004). However, with prolonged dry spells at the beginning of the rainy season, the risk of resowing and crop failures during the first stage of plant development is still a major concern in smallholder farming systems in SSA. Consequently, strategic agricultural decisions such as planting dates help reduce the need for crop resowing and crop failure and are, therefore, a key element in agricultural decision support. For farmers in SSA, crop planting date estimation, which is closely linked to the onset of the rainy season, is an important tactical operation as it determines the length of the plant growing period for the ongoing agricultural season. Accordingly, it is also related to the choice of crop and cultivar to plant.

Various definitions of the onset of the rainy season (ORS) in relation to the crop growing season have been developed for water-limited areas. Among them, rainfall-based approaches have been developed and are currently in use in SSA (e.g., Stern et al. 1981; Sivakumar 1988, 1990; Dodd and Jolliffe 2001; Chamberlin and Diop 2003; Laux et al. 2008). For these methods, rainfall amounts and wet- and dry-spell occurrences at the beginning of rainy season have been key variables in deriving the ORS and therefore suitable planting dates in SSA (Ati et al. 2002; Laux et al. 2008). Rainfall-based approaches are not crop specific, since information about crop type and phenology is not explicitly involved. But, they can be easily implemented and used for operational agricultural decision support.

With the increased development of process-based crop models in agricultural impact studies, new crop-specific approaches have been developed to estimate crop planting dates. These approaches have been used either at plot scale or regional scale and can be subdivided into two groups.

The first group consists of methods using only crop models to derive suitable planting dates. In this group, a crop yield optimization method is required (e.g., Stehfest et al. 2007). Depending on the crop model and the optimization method, this approach can be computationally time demanding. To overcome this issue, specific assumptions are usually made. For instance, Folberth et al. (2012) estimated crop planting dates by employing a crop model at a monthly or weekly time step. According to the region, they limited the planting date computation period by using a reported earliest and latest planting date. Although a time window of 1 month for crop planting is valuable in general, it is not favorable for regions in SSA where the growing season lasts only 3 months. In this first group, in addition to the high demand in computing time, crop models require a significant amount of input data. Therefore, this is a limitation for crop simulation, particularly in the data-scarce region of SSA.

The second group consists of a combination of crop models and rainfall distribution characteristics (e.g., Laux et al. 2010). In this approach, the first step is to derive planting dates that fulfill specific agronomical criterions using rainfall information only. Then, the resulting planting dates are used as input into a crop model to derive optimized planting rules by applying a suitable objective function and an optimization algorithm. This approach reduces significantly the required computation time and can be used to improve rainfall-based methods (Laux et al. 2010). This latter approach may open a new avenue in planting date estimates, since it can be used to derive crop and location-specific planting dates. However, determining the appropriate agrometeorological criteria to derive planting dates and the application of optimization methods to support agricultural decision making remain challenges.

This study fits into the second group. The research question is how to use crop planting date as an agricultural management strategy to support agricultural decision making in SSA. This research question is addressed by an approach aimed at optimizing crop and location-specific planting dates. For this purpose, fuzzy logic–based planting rules in combination with a large-scale crop model have been used. As a staple crop in Burkina Faso (Janin 2010), maize has been chosen as the target crop for simulation in this study.

The article is composed of three main parts. The first part deals with the study area, the input data, data processing, and the applied crop model [i.e., the General Large-Area Model for Annual Crops (GLAM)]. The second part deals with calibration of GLAM and the maize planting date optimization processes. The third section shows the results, followed by a discussion and our conclusions.

2. Study area

Burkina Faso (BF) is part of the West African Sahelian and Sudanian zones. It is a landlocked country stretching across 274 200 km^2 and lies between 9° and 15.5°N and between 6°W and 3°E (Fig. 1a). The country is mainly flat, with a mean altitude of about 300 m (Sivakumar and Gnoumou 1987). Approximately 90% of the population in BF lives in rural areas where rainfed crop production is the major source of food and income

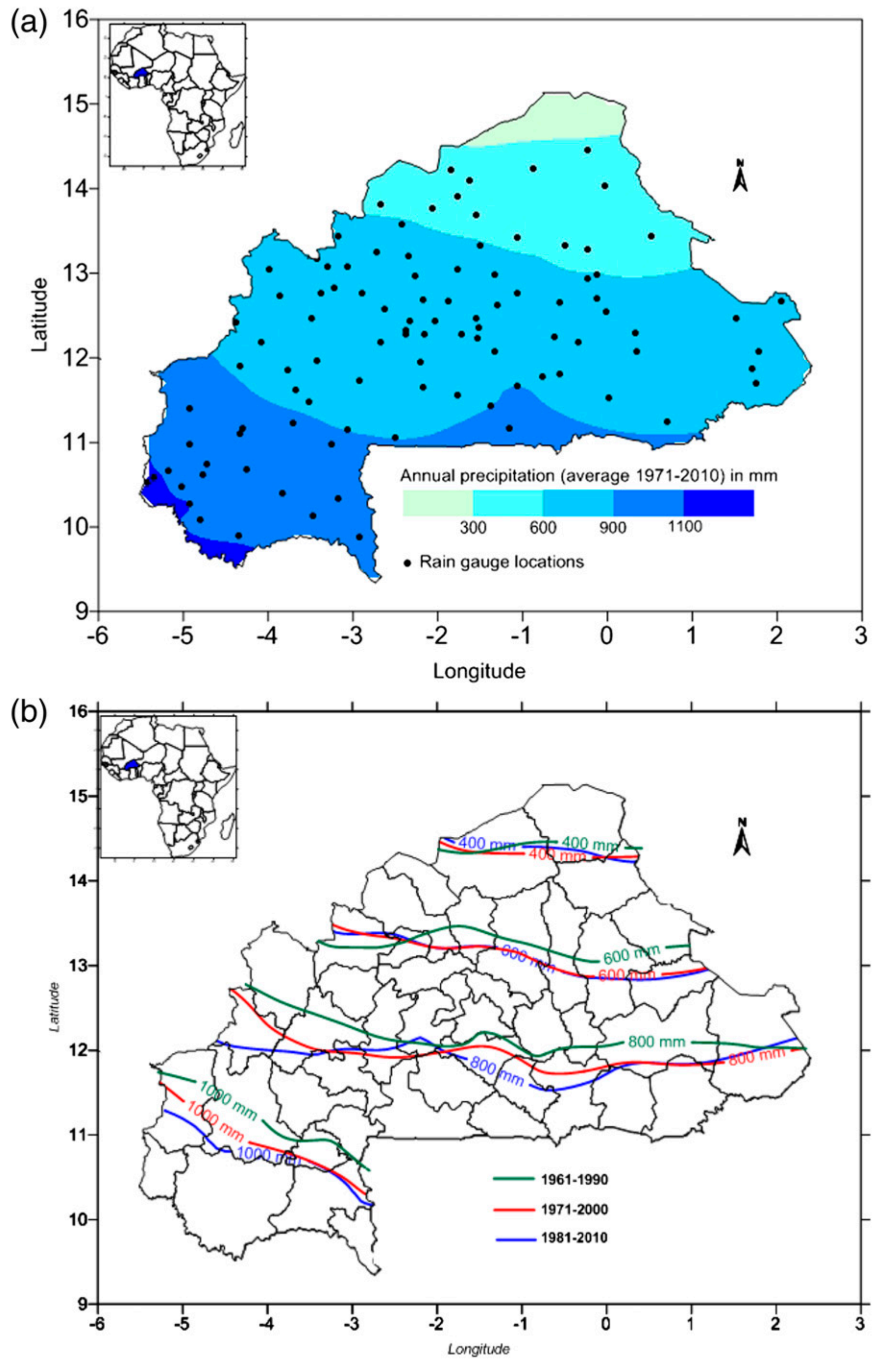

FIG. 1. (a) Mean annual precipitation (1971–2010) and rain gauge locations and (b) position of the running 30-yr mean isohyets in the study area. The insets show the location of BF within Africa.

(Badini et al. 1987). The main crops are sorghum (*Sorghum bicolor*), millet (*Panicum* sp.), and maize (*Zea mays* L.).

The climate of BF is characterized by two distinct seasons: a rainy season and a dry season. The dry season ranges from November to April and the rainy season ranges from May to October. During the dry season, the country is influenced by the Saharan anticyclone, which causes a flux of dry and cool air, the so-called Harmattan, over the country. The highest temperatures occur mainly in April–May while the coolest temperatures occur mainly in December–January (Sivakumar and Gnoumou 1987). At large scale, the rainy season is driven by the anomalies of the sea surface temperature (SST) in the tropical Pacific and Atlantic Oceans (e.g., Janicot et al. 1998;

Ward 1998). At regional scale, the rainfall variability across the country is influenced by the north–south fluctuation of the intertropical convergence zone associated with the West African monsoon (Sultan and Janicot 2000).

To capture rainfall variability in the study area, observed daily rainfall data provided by the Burkina Faso General Directorate of Meteorology (DGM) have been used. As shown in Fig. 1, the rainfall variability over the study area is great both spatially and temporally and is considered to be one of the most limiting factors in agriculture. On different time scales, a southward shifting of isohyets can be observed (Fig. 1b). The north–south rainfall gradient is more pronounced if compared with the east–west rainfall gradient. In Burkina Faso the mean annual rainfall decreases from more than 1100 mm in the southern part of the country to less than 300 mm in the northern part (Fig. 1a).

The mean temperature of the wet season has been estimated to range between 20° and 36°C and decreases from north to south across the country (Sivakumar and Gnoumou 1987). The agroecological zones match with the north–south distribution of the rainfall. The interannual and intraseasonal variability of rainfall is one of the major limiting factors of rainfed crop production in Burkina Faso.

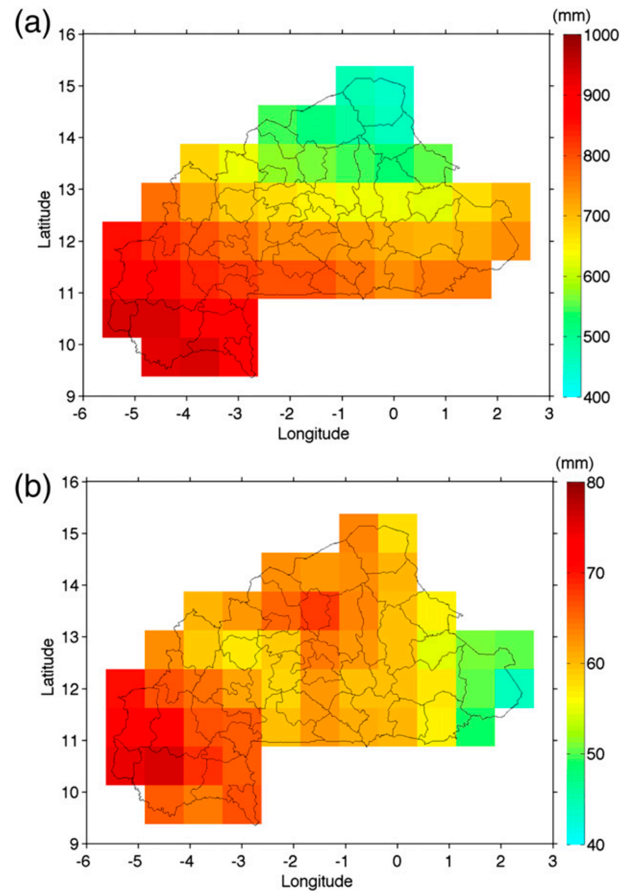

FIG. 2. (a) Gridded mean annual rainfall (1980–2010) and (b) gridded RMSE. RMSE between annual rainfall of any station and its corresponding grid cell is calculated and interpolated using OK.

3. Materials and methods

a. Climate data

The large area process–based model for annual crops (GLAM) requires daily weather data, mainly precipitation, mean temperature, and solar radiation (Challinor et al. 2004). Two sources of data have been used within the context of this study. Daily precipitation data from 141 rain gauges (Fig. 1a) have been provided by the DGM for a time series of 31 yr (1980–2010). These precipitation data have been gridded at a resolution of $0.75° \times 0.75°$ (i.e., 51 grid points for the study area) using ordinary kriging (OK). The OK technique is one of the most commonly used methods for interpolation. In this study, the number of rain gauges (141) was assumed to be acceptable for $0.75° \times 0.75°$ gridcell interpolation, using OK. The anisotropy of rainfall variability was well captured. Figure 2 illustrates the gridded mean annual precipitation (1980–2010) (Fig. 2a) as well as the error map (Fig. 2b).

For the study domain, European Centre for Medium-Range Weather Forecasts (ECMWF) Interim Re-Analysis (ERA-Interim) data (Dee et al. 2011) at a resolution of $0.75° \times 0.75°$ for minimum and maximum temperatures at 2 m above the surface and solar incoming shortwave radiation were retrieved from the ECMWF archive (http://data-portal.ecmwf.int/data/d/interim_daily/). We retrieved daily data from 1 January 1980 until 31 December 2010. A preprocessing of the ERA-Interim data was performed to fit the format, units, and time scale required by GLAM.

b. Soil data

Gridded soil types and their hydrological properties (soil water content at saturation and soil water content at field capacity, soil water content at the wilting point) have been derived from the Harmonized World Soil Data (HWSD) dataset in combination with ArcInfo and a soil water content computation algorithm. First, based on the climate data grid coordinates for the target area, the matching soil mapping unit was derived using a geographic information system (ArcInfo). Then, the dominant soil type for each grid location, and its associated soil properties, are summarized using the tabular soil database from HWSD (FAO 1991). Finally, the soil water content parameters were estimated following an algorithm designed for the computation of soil water limits (Ritchie et al. 1999; Suleiman and Ritchie 2001).

c. Crop yield data

Province-level maize yields (kg ha^{-1}) over 27 yr (1984–2010) from all 45 provinces in Burkina Faso have been collected. Two datasets of maize yield have been provided by the AGRHYMET Regional Centre and the BF National Agricultural Statistic Division. These datasets contain annual rainfed crop production and estimated land area allocated to grow maize. The country's database of crop production and estimated land area originate from survey campaigns with farmers every year during harvest time. Yearly provincial crop yields (kg ha^{-1}) have been computed from both crop production data and the estimated land area. To identify a reliable period for the calibration of the crop model, the two datasets were analyzed in terms of similarity. This is done for each province and each year separately. The ratio of missing data has also been evaluated for each provincial dataset. Based on the coincidence of both datasets and the experiences from the data providers, the period 2000–10 has been selected and used in this study. The differences between the two datasets were less than 5% with no missing data for the period 2000–10. Further, the province-level maize yield data from 2000 to 2010 have been transformed into gridded data at a resolution of 0.75° × 0.75° to fit the weather and soil data grids. As no significant trends in crop yield (e.g., imposed by ever-increasing technological development) could be detected for the period 2000–10, no detrending has been applied. The gridded crop yield has been calculated by using a composite weighted average for all provinces that share the same grid. The land areas of the concerned provinces have been used as weights:

$$Y_{\text{grid}}\,(\text{kg ha}^{-1}) = \frac{\sum_{i=1}^{n} w_i \times Y_{\text{district}(i)}\,(\text{kg ha}^{-1})}{\sum_{i=1}^{n} w_i}, \quad (1)$$

where Y_{grid} (kg ha^{-1}) is the gridded crop yield, $Y_{\text{district}(i)}$ is the crop yield in district i, w_i is the fraction of land area of district i within the grid cell, and n is the number of grid cells that share the land area of the grid cell.

d. Large-scale crop model GLAM

Spatiotemporal variability in crop yields is associated with climate variability. For the first studies attempting to link climate to agriculture outputs, statistical tools were used to derive quantitative or qualitative relationships between crop production and climate variables such as precipitation and temperature. Nowadays, efforts are being made to describe the dynamic relationship between crop production and climate by using process-based crop models (Robert and Bruce 1998; Wallach et al. 2006). Most crop models have been designed to be used at plot scale and therefore specific assumptions have to be made to upscale results to larger scales (Hoogenboom 2000). In recent years, large-scale process-based crop models are increasingly designed and being used in the analysis of regional agricultural production systems (e.g., Moen et al. 1994; Brock and Brink 1996; Challinor et al. 2005; Tao et al. 2009). In this study, GLAM (Challinor et al. 2004) has been used. The GLAM has been designed to simulate the impact of climate on crop yield or biomass. As a process-based crop model, it simulates crop growth and development with daily time step. This crop model operates on spatial scales commensurate with those of global and regional climate models (Challinor et al. 2004). It can be used to assess the impacts of climate variability and change on annual crop yields. GLAM was initially calibrated and validated for groundnut production in India with the potential to be applied to a large range of crops, as the crop growth processes are generic. In water-limited crop production regions, GLAM has been shown to be able to capture the strong relationship between weather and crop production (Challinor et al. 2004). To simulate a crop growing season, GLAM requires mainly daily time series of precipitation, temperature, and radiation as weather inputs. In GLAM, the accumulated aboveground biomass is converted into crop yield using the harvest index when the harvest time is reached. The simulated daily transpiration and the crop transpiration efficiency parameter are used to compute the daily biomass accumulation from crop emergence to maturity. A harvest index rate parameter is used to increase the harvest index from 0 to a maximum value during the grain-filling and maturity stages.

In GLAM, the yield responses to, for example, fertilizer, plant population density, and pest and diseases separately, have not been explicitly formulated. A unique parameter called yield gap parameter (YGP), which is location specific, is used to take account of yield losses due to the mean effects of nutrient deficiency, nonoptimal management, pest and diseases incidence. For any specific location, the YGP is calculated by minimizing the root-mean-square error (RMSE) between simulated yield and observed yield using all potential values of YGP. For more details on the dynamic processes in GLAM, the reader is referred to Challinor et al. (2004).

In this study, GLAM has been calibrated for the first time for maize growing in Burkina Faso and is used to derive optimized maize planting dates.

e. GLAM calibration

GLAM has been calibrated for maize in Burkina Faso using a genetic algorithm optimization method. Genetic algorithms (GAs) were first introduced by Holland (1975). GAs are heuristic methods inspired by natural evolution. They mimic key operators of natural evolution such as genetic recombination (crossover) and mutation. These algorithms encode a potential solution to a specific problem in a simple chromosome-like data structure and apply recombination and mutation operators to these structures so as to preserve critical information. For instance, a string of bits, encoding each parameter of a potential solution can be seen as a gene in a chromosome while the concatenation of such strings is comparable to a chromosome in genetics.

The capability of GAs to approach (and eventually to find) the global optimum in an optimization problem is based on the choice of reproduction operators, their appropriate representation, and the formulation of the objective function [the so-called fitness function; Sivanandam and Deepa (2008)]. The latter is specific to the problem that one is dealing with in terms of the objective to be reached.

The first step in the implementation of any genetic algorithm is to generate an initial population that consists of random selections of potential solutions in the parameter space. In this study, a binary encoding is used to encode each member of the population as a binary string of length $p \times 2^n$, where p denotes the number of parameters to be calibrated in the GLAM and n denotes the number of bits (2^n is the number of possible values for a given parameter) (Carroll 1996a,b).

In GLAM, a total number of 32 parameters have been calibrated for maize for 85–100 days of a growing period, representing the most dominant group of maize cultivars in BF (Sanon and Dembélé 2001): phenology parameters [base temperature, optimum temperature, maximum temperature, growing degree days (GDDs)], biomass parameters [temperature efficiency (TE), harvest index, maximum value of normalized TE], evapotranspiration parameters (evaporation coefficient, maximum value of potential transpiration, vapor pressure deficit, soil heat flux coefficient), leaf area index (LAI) parameters (critical LAI, daily maximum value of LAI, extinction coefficient, soil water fraction for reduced LAI growth), drainage and uptake parameters (uptake diffusion coefficient, root length density), and soil parameters (albedo, depth of soil over which evaporation occurs, extractable front velocity).

The GDD range for each crop development stage is crucial for the simulation, since the crop phenology and growing period heavily depend on it. To deal with the GDD variability in the target area, the 85–100-day growing period of the maize crop have been transformed into GDDs considering four maize growth stages (vegetative growth, flowering, grain filling, and maturity). The range of GDDs for each development stage has been computed using daily mean temperatures for the target area and crop phenological base temperatures T_B. We have chosen T_B to be in the range of 8°–14°C (Birch et al. 1998). The GDDs have been calculated for each grid cell and for each crop development stage. Then, the computation of the GDD mean value (GDD_m) and GDD standard deviation (GDD_{std}) for each development stage is performed over the target area. Finally, assuming a normal distribution, a GDD ranging from $GDD_m - 2 \times GDD_{std}$ to $GDD_m + 2 \times GDD_{std}$ is set for each development stage of maize crop. For the other parameters, the selected range has been taken from GLAM's generic parameters file (http:/www.see.leeds.ac.uk/research/icas/climate_change/glam/glam.html) and from the literature (Carberry et al. 1989; Muchow and Carberry 1989; Carberry 1991; Birch 1996; Maddonni and Otegui 1996; Birch et al. 1998; Rasse et al. 2000; Sanon and Dembélé 2001; Sanon et al. 2002).

In addition to the 32 parameters, planting dates are needed to perform crop simulations with GLAM. Planting dates can be set for the simulation in two different ways. Either observed planting dates or computed planting dates can be used as input into GLAM. Observed planting date data are usually not available for large-scale analyses in SSA. Therefore, estimated planting dates have been used. To estimate planting dates, the GLAM intrinsic function can be employed. For GLAM calibration purposes, the GLAM intrinsic function has been replaced by a crop-specific soil water balance module for planting date computation. This water balance module uses the water balance module of GLAM. It computes daily soil water balance for the first vegetative growth phase of the maize crop, considering daily rainfall, soil characteristics, and simulated maize daily actual evapotranspiration. After 1 May, the crop-specific soil water balance has been computed on a daily basis. The estimated planting date is set to be the first day between 1 May and before 31 July for which the crop-specific soil water balance is greater than zero for each day in the following 30 days. The planting date is set to 31 July if no planting date is found in the aforementioned period. This soil water balance algorithm should mimic the traditional planting behavior of smallholder farmers in SSA. The resulting planting dates are not optimal in terms of crop yield. Indeed, they have the potential to

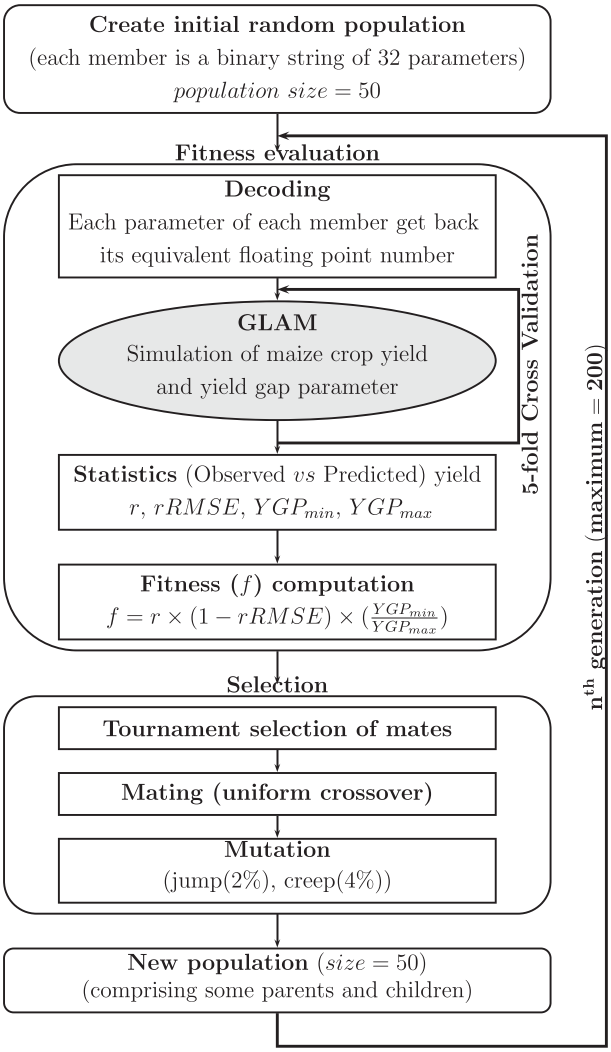

FIG. 3. Flowchart of GLAM calibration for the maize crop using a GA.

avoid crop failure and not to reach optimum crop yield. These dates are used as planting dates for the calibration of GLAM.

The different steps in the process of GLAM calibration for maize crops using a GA have been summarized in Fig. 3. For optimization purposes, the objective function in the GA has been formulated in a way such that it captures the degree of coincidence between simulated and observed maize yields for the calibration period. In addition, the variability in YGP is part of the objective function. For a specific location, we assumed that the best setting of parameters in GLAM for the target grid cell should result in a high positive correlation r, a low RMSE, and low variability in YGP. A k-fold cross validation is used to overcome the limited size of the calibration period (2000–11). To ensure the robustness of the calibrated parameters and to reduce the computation time, the value of k is set to 5 in this study. In the process of fivefold cross validation, the 11-yr-period data are randomly partitioned into five complementary subsets (one subset of 3 yr and four subsets of 2 yr). Out of five, four randomly chosen subsets are used as training sets and the remaining subset is used for validation. Fifty loops (10 initializations × 5 combinations of training–validation sets) of cross validation are performed. Finally, an average over loops is used to compute r and RMSE. The minimum and the maximum of YGP over all loops are retained. Thus, for each setting of parameters in GLAM, a fitness value is computed after the fivefold cross validation. The highest fitness value should correspond to the best set of parameters. To fulfill these requirements, the fitness function is defined as

$$f() = r(1 - \mathrm{rRMSE})\left(\frac{\mathrm{YGP}_{\mathrm{min}}}{\mathrm{YGP}_{\mathrm{max}}}\right), \qquad (2)$$

where r denotes the Pearson correlation coefficient between the simulated and observed yields, rRMSE is the relative root-mean-square error, and $\mathrm{YGP}_{\mathrm{min}}$ and $\mathrm{YGP}_{\mathrm{max}}$ denote the minimum and maximum values of the yield gap parameter, respectively.

f. Fuzzy logic approach for crop planting date estimation

The term fuzzy logic emerged in the development of the theory of fuzzy sets by Zadeh (1965). It refers to the principles and methods of representing knowledge that employs intermediate truth values. Fuzzy logic provides a way to represent subjective attributes of real-world problems in computing (Belohlavek and Klir 2011).

Optimized maize crop planting dates have been derived from rainfall time series data using a fuzzy logic approach in combination with GA. For agronomists, wet conditions are crucial after the planting date. They are necessary to ensure crop emergence and an optimum first-stage development. During the first stage of crop development, the root system of the crop is still not well developed enough to cope with longer dry spells. Therefore, crop failure and resowing might be avoided if wet conditions during the first vegetative growth stage occur. The rainfall-based estimation of planting dates for agricultural decision support uses threshold values for relevant agrometeorological variables such as rainfall amount and the number of wet- and dry-spell lengths, for a given period. However, the uncertainties due to the limited number of observations and measurement errors have to be taken into account when dealing with hydrometeorological variables. To cope with rainfall data uncertainties and the vagueness around the explicit value of these variables, a fuzzy logic–based approach has been used to compute optimized planting dates and for improved crop production (Laux et al. 2010, 2008). This approach uses the concept of fuzzy logic membership functions to deal with the cumulative rainfall amount and the wet- and dry-spell lengths.

Following Laux et al. (2008, 2010), three fuzzy functions γ_1, γ_2, and γ_3 for cumulative rainfall amount within a 5-day spell, the number of rainy days within a 5-day spell, and the longest dry-spell length in the next 30 days after the planting day, respectively, have been defined (Fig. 4). The variables a_1 and a_2 of the membership γ_1 vary between 10 and 30 mm, b_1 and b_2 of the membership γ_2 vary between 1 and 5 days, and c_1 and c_2 of membership γ_3 vary between 5 and 10 days. The defuzzification parameter k varies between 0.1 and 1. Using a list of if–then clauses, γ_1 is set to 0 if the 5-day cumulative rainfall is less than a_1 mm and 1 if the 5-day cumulative rainfall is greater or equal to a_2 mm. For a 5-day cumulative rainfall ranging between a_1 and a_2, the value of γ_1 is obtained by a linear interpolation between a_1 and a_2. Similarly, γ_2 and γ_3 are computed based on their specific parameters.

The GA, coupled with this fuzzy logic approach and GLAM, calibrated for maize, has been used to derive 10 ensemble members that are composed of optimized sets of fuzzy parameters (a_1, a_2, b_1, b_2, c_1, c_2, k). The flowchart of the respective process is illustrated in Fig. 5.

The optimized fuzzy parameters are crop and location specific. For the optimization process, a fitness function is defined to discriminate among the different sets of parameters in terms of performance. The objective is to optimize planting dates so that they increase crop production and also reduce the coefficient of variation (CV). Therefore, the fitness function is defined as

$$f() = \frac{1}{\mathrm{CV}}. \tag{3}$$

For a specific location, the optimization process yielded a set of optimum fuzzy parameters. From this set, an ensemble of 10 members is retained. The 10 ensemble members consist of parameter sets, which result in high crop yields and a low variability of simulated crop yield (i.e., high fitness) over time. Using a time series of rainfall of a specific grid cell with the ensemble of optimized fuzzy parameter sets, an ensemble of optimized planting dates for maize has been computed by applying the proposed fuzzy logic approach algorithm. The flowchart in Fig. 6 illustrates the individual steps.

g. *Evaluation of planting dates*

Optimized planting dates (OPDs) are computed using the derived optimum fuzzy parameters in combination

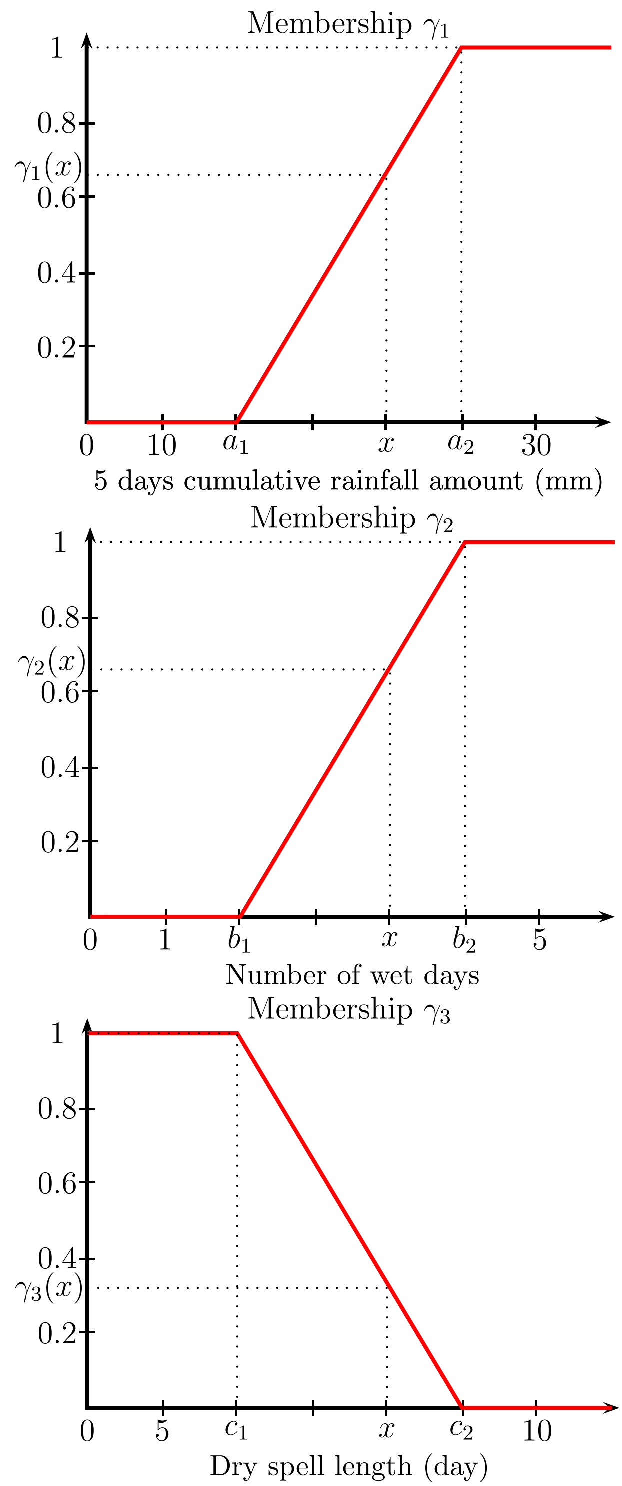

FIG. 4. Fuzzy logic memberships of (top) rainfall amount, (middle) number of wet days, and (bottom) dry-spell length.

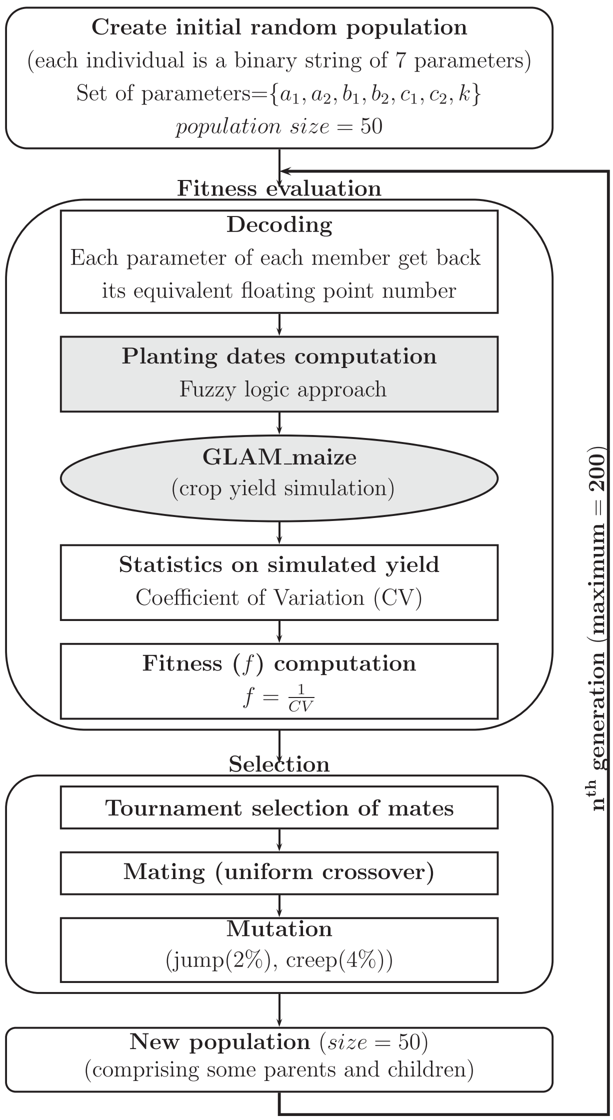

FIG. 5. Flowchart of planting date optimization using a GA.

with daily rainfall time series. To evaluate the efficiency of the OPDs, two well-known and regionally established approaches have been used to calculate planting dates for comparison. These two approaches are as follow:

(i) Diallo (2001)—the date after 1 May, when at least 20 mm of rainfall accumulates over three consecutive days and when no dry spell of more than 10 days occurs within the next 30 days; this approach is currently used at the AGRHYMET Regional Centre in Niamey, Niger; and

(ii) Dodd and Jolliffe (2001)—the first day of a spell of 5 days in which at least 25 mm of rain falls, on condition that no dry period of more than 7 days occurs in the following 30 days; this approach is currently in operation as an agricultural decision support tool at the Burkina Faso Directorate General of Meteorology.

A deviation of planting dates and a relative deviation of maize mean yield are used to compare the different approaches. The deviation of planting date (D_{PD}) is calculated as

$$D_{\mathrm{PD}}(\mathrm{days}) = \mathrm{OPD} - P_D, \tag{4}$$

where P_D is the planting date based on either Diallo (2001) or Dodd and Jolliffe (2001).

The relative deviation of the mean maize yields (D_{yield}) is given as

$$D_{\mathrm{yield}}(\%) = 100 \times \frac{(\mathrm{YIELD_{OPD}} - \mathrm{YIELD})}{\mathrm{YIELD}}, \tag{5}$$

where YIELD is the mean yield either based on Diallo (2001) or Dodd and Jolliffe (2001), We denote the mean yield based on OPD as $\mathrm{YIELD_{OPD}}$.

4. Results

a. GLAM calibration for maize in Burkina Faso

Since GLAM has not yet been calibrated for maize in West Africa, a GA-based calibration has been performed. A summary of the range of variability of calibrated parameters in the study area is presented in appendix A. The performance of the calibration has been evaluated using Pearson's correlation coefficient (r) and the RMSE between the simulated and observed yields over the period 2000–10. Figure 7a depicts the location-specific r over Burkina Faso. The minimum r is 0.6, while r is larger than 0.75 for 80% of all locations (41 of 51 grid cells). In the majority of locations, the calibrated GLAM is able to capture 50% ($R^2 = 0.5$) of the linear variability of the maize crop yield for the period 2000–10. At the significance level $\alpha = 0.05$, the r values are statistically significant (Fig. 7b) for all locations. Figure 7a reveals a distinct homogeneous high correlation ($r \geq 0.8$) in the southwest of BF. The rRMSE, shown in Fig. 7c is less than 50% for all locations. The simulated maize crop yield in the majority of locations deviates from the respective observed maize crop yield by less than 25% and even less in the southwest of BF. According to Fig. 7, it is evident that the performance of the calibrated GLAM simulation for maize clearly depends on the specific location.

b. Maize-optimized planting dates and yield

A 10-member ensemble of fuzzy logic parameter sets is used to derive OPDs over the period 1980–2010. The

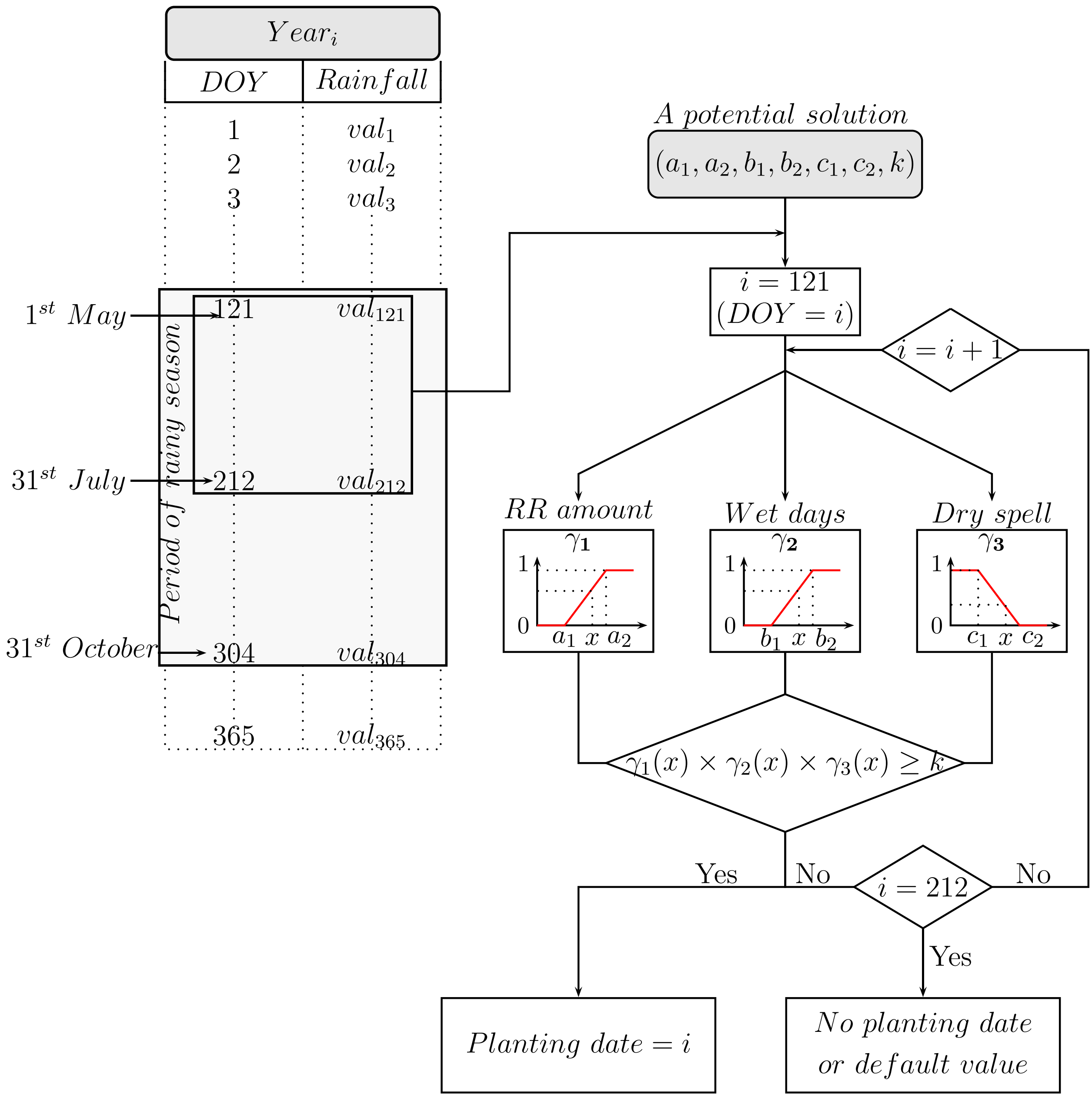

FIG. 6. Flowchart of planting date computation based on daily rainfall. The box spanning the period between 1 May and 31 Jul represents the potential crop planting window within the rainy season (1 May–31 Oct) in SSA.

ensemble mean values of the fuzzy logic parameters are presented in appendix B. The results shown in Fig. 8 depict (a) the mean OPD ($\overline{\mathrm{OPD}}$) and (b) the standard deviation of OPD (σ_{OPD}) for a sample of 310 (10 members $\times$ 31 yr) optimized planting dates for each grid cell. Between 7 May and 5 July $\overline{\mathrm{OPD}}$s vary across the country following a north–south gradient. In general, the earliest OPDs occur in May in the southern part of BF, whereas the latest OPDs occur in June–July in the northern part of the country. Following a similar spatial pattern, σ_{OPD} varies between 2 and 18 days. The variability of OPDs is greater in the northern than in the southern parts of the country.

The OPDs have been used as input in GLAM to simulate maize yield. Figure 8c shows the spatial distribution of mean maize yield over the period 1980–2010. The mean yield varies between 500 and 3000 kg ha^{-1} with the highest (lowest) yields in the southernmost (northernmost) parts of BF. The highest simulated mean yields can be found in southwestern Burkina Faso,

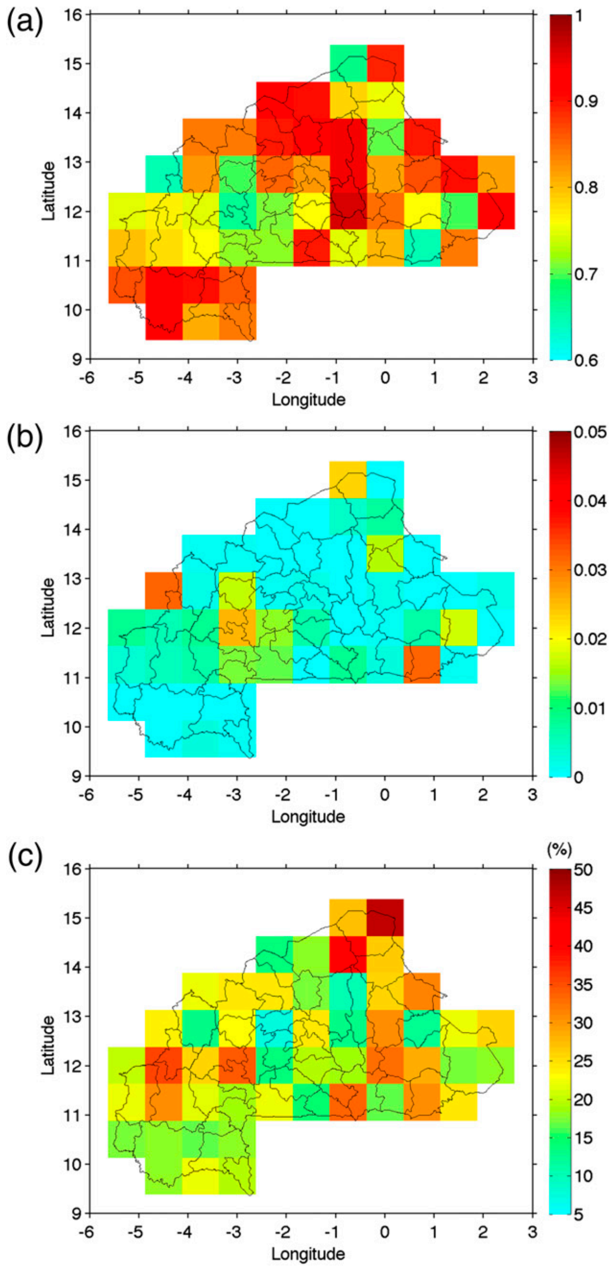

FIG. 7. Performance measures of GLAM calibration: (a) Pearson correlation coefficient between observed and simulated maize crop yield, (b) p value of Pearson's correlation coefficient, and (c) relative RMSE between observed and simulated maize yields.

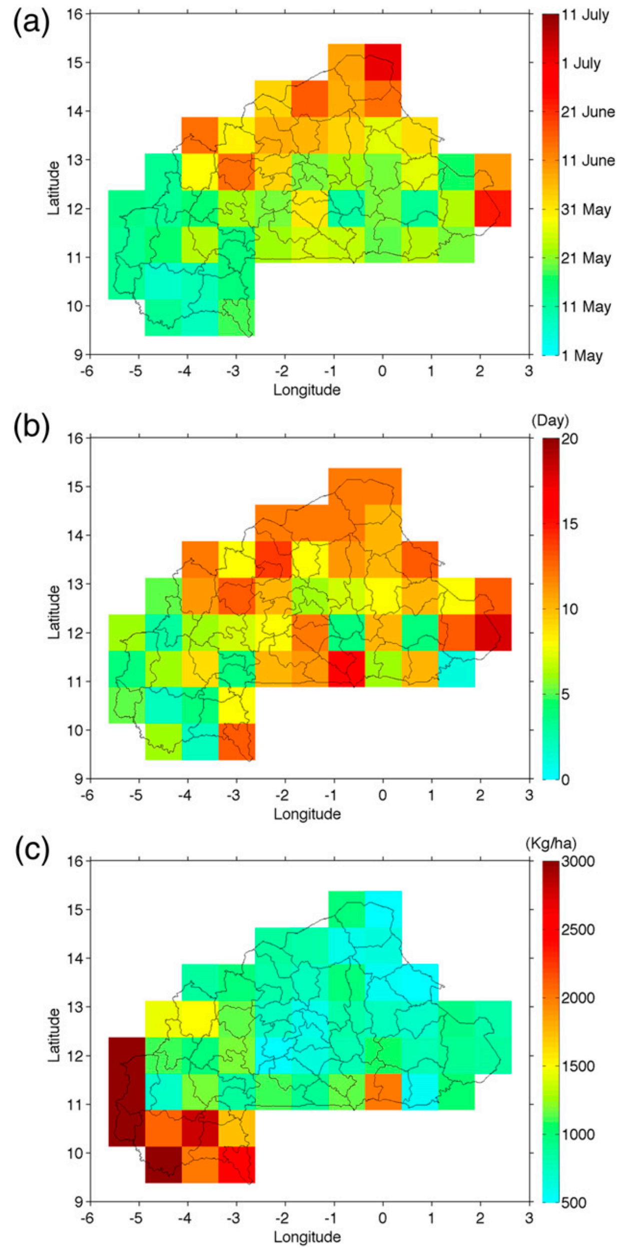

FIG. 8. Maize OPDs and simulated maize yield in BF for the period 1980–2010: (a) mean OPDs, (b) standard deviation of OPDs, and (c) mean simulated maize yield using OPDs.

whereas yields are less than 2000 kg ha^{-1} for the central and northern parts of the country.

c. Comparative analysis of planting date approaches

Planting dates and resulting simulated maize yields are computed for the approaches of Diallo (2001) and Dodd and Jolliffe (2001), and then compared to the OPD approach. On average, the deviation in planting dates between the OPD approach and the approaches of Diallo (2001) and Dodd and Jolliffe (2001) varies between −20 and +12 days for both Diallo (2001) (Fig. 9a) and Dodd and Jolliffe (2001) (Fig. 9b). The lowest (highest) deviation magnitude is mainly located in the southwestern (northern) part of BF. In general, the OPD approach yielded the earliest planting dates if compared to the planting dates computed by the approaches of Diallo (2001) and Dodd and Jolliffe (2001).

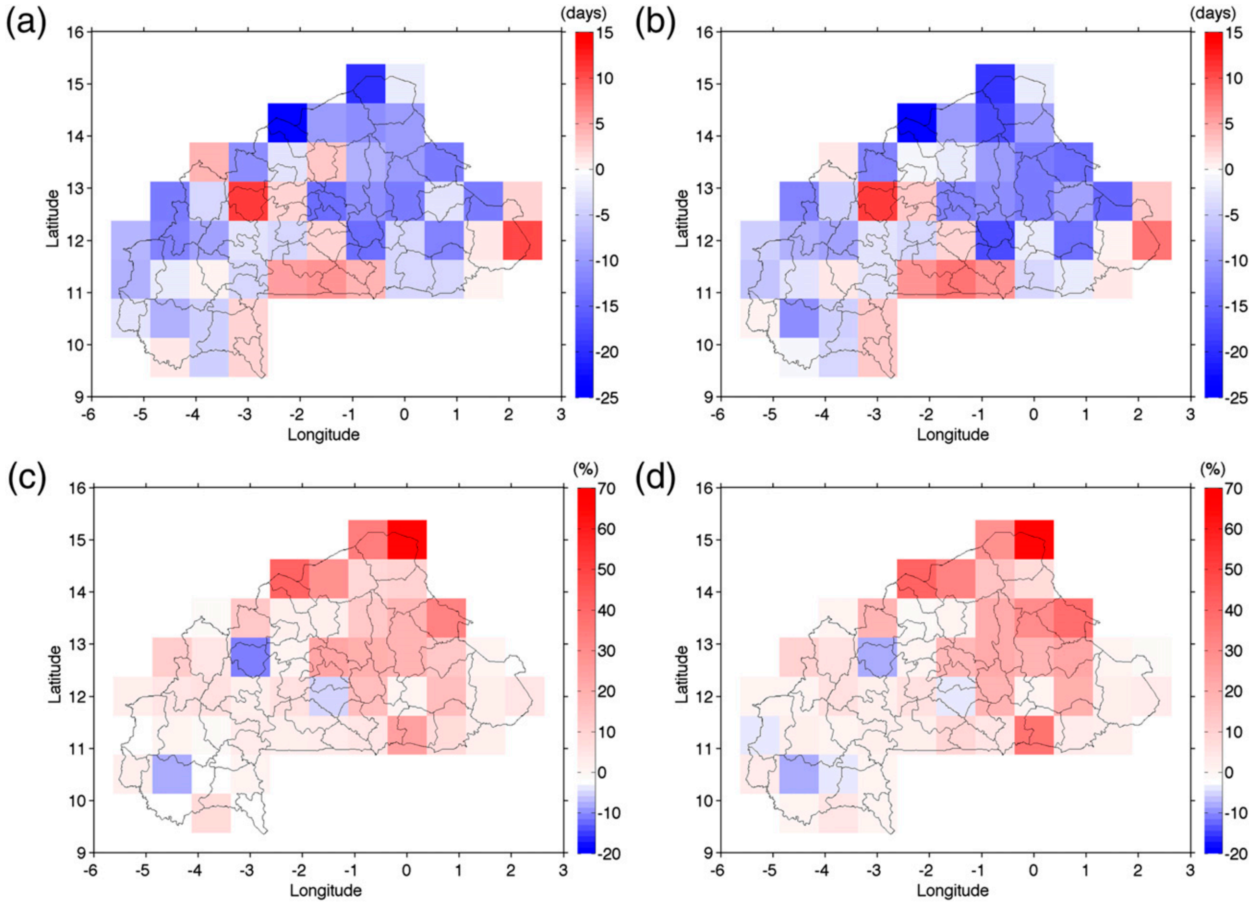

FIG. 9. Comparison of (top) planting dates and (bottom) simulated yield obtained by OPDs and the approaches of (left) Diallo (2001) and (right) Dodd and Jolliffe (2001) for maize cultivation in BF: (a) planting date deviations [OPD − Diallo (2001)], (b) planting date deviations [OPD − Dodd and Jolliffe (2001)], (c) relative deviation of mean maize potential yield [OPD − Diallo (2001)], and (d) relative deviation of mean maize potential yield [OPD − Dodd and Jolliffe (2001)].

The deviation of maize potential yield ranges between −10% and +60% while positive values prevail (Figs. 9c and 9d). Except for the southern part, the potential yield obtained by OPDs results in an increase of at least 10% in mean yield relative to those obtained by Diallo (2001) and Dodd and Jolliffe (2001). For the southern part of the country, however, this increase in mean yield is less pronounced.

5. Discussion and conclusions

An approach to objectively derive crop planting dates is presented and applied for the first time to maize cultivation in West Africa. The approach accounts for crop-specific meteorological and soil requirements during the whole growing period. The results show that the optimized planting dates generally follow the prevailing north–south gradient of rainfall with earlier (later) planting in the south (north). This gives evidence that planting dates depend strongly on location. This finding is in agreement with studies of Kniveton et al. (2009) and Laux et al. (2010), who account for local and regional differences, respectively. The OPD approach is similar to the approach of Laux et al. (2010). Instead of using a crop model designed to work on a local scale, the regional crop model GLAM is used. A genetic algorithm is used to derive robust planting rules at a regional scale, which significantly reduces the required iterations, and thereby computing time.

For SSA, several methods of estimating the onset of the rainy season are in operation, giving recommendations for planting dates. These approaches are usually applied at the local scale. At the BF National Meteorological Services and the AGRHYMET Regional Centre, the approaches of Diallo (2001) and Dodd and Jolliffe (2001), which are regionally adapted versions of Stern et al. (1981, 1982), are currently in operation in support of agricultural decision making in SSA. For the southeast of Burkina Faso, the OPD reaches a similar level of performance in terms of potential yields compared to the two well-established methods; that is, these approaches are already well adapted for this intensively used and maize-dominated agricultural region.

In comparison with these approaches in operation, the proposed OPD has the following advantages:

(i) Once a calibrated process-based crop model is available, agrometeorological and crop yield data are required to derive crop and location-specific planting rules and to estimate planting dates. Besides the required knowledge needed to calibrate the crop model, this approach can be seen as fully objective. However, agronomic and agrometeorological knowledge is still required to validate the outcome of this study.

(ii) Instead of relying exclusively on rainfall amount and distribution around planting, the OPD approach not only accounts for plant water requirements and availability throughout the whole growing period, but also for radiation and temperature. This information is inherently included by coupling the planting rules to a process-based crop model.

(iii) The use of fuzzy logic to estimate planting rules instead of binary logic gives further flexibility in estimating reliable planting dates where strict thresholds may fail. This is exemplarily illustrated for the amount of rainfall in a 5-day spell. A strict value of, for example, 25 mm, as used in the approach of Dodd and Jolliffe (2001), would exclude a reasonable planting date in which, for instance, 24.9 mm of rain are recorded, even if significant rain and favorable conditions for crop growth follow.

(iv) Finally, the OPD approach is not elaborating a single specific planting date, but rather it is suggesting a set of reasonable planting rules, leading to a time window for planting of approximately 2 weeks. This can help to increase the adoptability of this approach for smallholders, because their decisions about planting also depend on other external factors such as the availability of seeds, labor, machinery, etc.

This approach achieves higher potential yields across BF compared with the methods currently in operation. Detailed in-field validation is required before being implemented at agricultural national and regional centers. Further studies will be conducted in order to evaluate the potential benefits of the OPD approach if combined with improved seasonal climate predictions accounting for the intraseasonal rainfall variability.

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APPENDIX A

Summary of the Range of Variability of GLAM Calibrated Parameters

See Table A1 for a summary of the range of variability of calibrated parameters in the study area.

TABLE A1. Summary of the range of variability of GLAM-calibrated parameters.

Parameter	Min	Max
Base temperature for all development stages (°C)	8	14
Optimum temperature for all development stages (°C)	25	35
Max temperature for all development stages (°C)	35	45
GDDs from emergence to anthesis (°C)	500	1000
GDDs from anthesis to grain filling (°C)	300	8000
GDDs from grain filling to max LAI (°C)	150	300
GDDs from max LAI to maturity (°C)	200	500
Max LAI growth ($m^2\,m^{-2}\,day^{-1}$)	0.01	0.21
Constant of soil heat flux (–)	0.1	0.8
Extinction coef for PAR (–)	0.1	0.9
Soil water content fraction threshold (–)	0.3	0.8
Extractable front velocity ($cm\,day^{-1}$)	0.2	1.1
Depth of soil over which evaporation occurs (mm)	20	50
Albedo (–)	0.1	0.3
Uptake diffusion coef ($cm^2\,day^{-1}$)	0.3	0.7
LAI corresponding to max transpiration (-)	0.6	2.4
Max of potential transpiration (cm)	0.4	0.6
Vapor pressure deficit (kPa)	0.6	1.1
Transpiration efficiency (Pa)	1	4

APPENDIX B

Mean Values of Optimized Fuzzy Parameters Set

See Table B1 for a presentation of the ensemble mean values of the fuzzy logic parameters.

TABLE B1. Mean values of optimized fuzzy parameters set.

Lat (°)	Lon (°)	a_1 (mm)	a_2 (mm)	b_1 (day)	b_2 (day)	c_1 (day)	c_2 (day)	k
15.00	−0.75	11	13	2	3	7	9	0.70
15.00	0.00	12	18	2	3	7	8	0.60
14.25	−2.25	14	24	3	5	7	9	0.80
14.25	−1.50	11	14	2	4	7	9	0.70
14.25	−0.75	11	16	2	4	7	9	0.70
14.25	0.00	14	21	2	4	6	8	0.60
13.50	−3.75	13	20	2	3	7	9	0.60
13.50	−3.00	10	13	2	3	7	9	0.70
13.50	−2.25	13	16	2	3	7	8	0.60
13.50	−1.50	19	23	2	3	7	9	0.80
13.50	−0.75	11	13	2	4	7	9	0.80
13.50	0.00	10	14	2	3	7	9	0.70
13.50	0.75	11	13	2	3	7	9	0.80
12.75	−4.50	12	15	2	3	7	9	0.80
12.75	−3.75	11	13	1	3	7	8	0.80
12.75	−3.00	21	25	2	3	7	9	0.80
12.75	−2.25	18	26	3	4	6	9	0.80
12.75	−1.50	11	15	3	4	6	9	0.60
12.75	−0.75	11	14	2	3	7	9	0.60
12.75	0.00	11	14	2	4	6	8	0.70
12.75	0.75	10	13	2	4	6	8	0.80
12.75	1.50	12	18	2	3	7	9	0.50
12.75	2.25	18	23	2	3	7	8	0.70
12.00	−5.25	12	18	1	3	6	9	0.70
12.00	−4.50	11	14	2	3	6	9	0.70
12.00	−3.75	11	15	2	3	7	8	0.60
12.00	−3.00	12	18	1	3	7	9	0.60
12.00	−2.25	10	13	2	3	7	9	0.80
12.00	−1.50	12	16	3	4	7	8	0.70
12.00	−0.75	12	16	2	3	7	9	0.60
12.00	0.00	15	23	1	3	6	8	0.60
12.00	0.75	11	14	1	3	6	8	0.60
12.00	1.50	13	19	2	3	7	9	0.80
12.00	2.25	17	24	2	4	8	9	0.90
11.25	−5.25	15	23	2	3	7	9	0.50
11.25	−4.50	13	20	3	4	7	9	0.50
11.25	−3.75	19	25	2	4	7	9	0.70
11.25	−3.00	12	18	3	4	6	9	0.60
11.25	−2.25	15	20	2	4	6	8	0.80
11.25	−1.50	18	25	2	3	7	9	0.80
11.25	−0.75	17	22	2	4	7	9	0.70
11.25	0.00	11	16	2	3	6	8	0.70
11.25	0.75	11	17	2	4	6	8	0.60
11.25	1.50	15	19	2	4	6	8	0.90
10.50	−5.25	17	22	2	3	6	8	0.70
10.50	−4.50	14	20	2	3	6	9	0.60
10.50	−3.75	11	16	3	4	7	8	0.70
10.50	−3.00	15	23	3	4	7	8	0.80
9.75	−4.50	17	24	2	3	7	9	0.60
9.75	−3.75	11	16	4	5	6	9	0.80
9.75	−3.00	15	22	2	3	6	8	0.70

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